The PEGASOS Single-Station Sigma Model: Focus on Databases

Adrian Rodriguez-Marek
Virginia Tech

with contributions from

Outline

• Single-station sigma: overview
• Requirements for use of single-station sigma
• Proposed model: Databases
• Proposed Phi (intra-event) Model
• Conclusions
Single-station sigma: overview

• A Ground Motion Prediction Equation (GMPE)

\[
\ln(y) = f(M, R, \text{Site, etc...}) + \Delta
\]

\[
\ln(y) : \text{median prediction}
\]

\[
\Delta : \text{residual}
\]

\[
\text{mean}(\Delta) = 0
\]

\[
\text{stdev}(\Delta) = \sigma_{tot}
\]
Breakdown of total sigma

\[ \Delta = \delta W_{es} + \delta B_e \]

<table>
<thead>
<tr>
<th>Residual component</th>
<th>Standard deviation</th>
<th>Required Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta B_e ): event term</td>
<td>( \tau ) (aleatoric)</td>
<td>Global: Multiple recordings (surface) at different sites from EQ in multiple source regions</td>
</tr>
<tr>
<td>( \delta W_{es} ): within-event residual</td>
<td>( \phi ) (aleatoric)</td>
<td></td>
</tr>
</tbody>
</table>

\[ \sigma_{tot} = \sqrt{\phi^2 + \tau^2} \]
\[ \Delta = \delta W_{es} + \delta B_e \]

From Al Atik et al. (2010)
Further breakdown

$$\delta W_{es} = \delta S2S_s + \delta WS_{es}$$

<table>
<thead>
<tr>
<th>Residual component</th>
<th>Standard deviation</th>
<th>Required Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta S2S_s$: site-term</td>
<td>$\phi_{S2S}$</td>
<td>Site specific:</td>
</tr>
<tr>
<td></td>
<td>(epistemic)</td>
<td>• Multiple recordings (surface) at different sites from EQ in multiple source</td>
</tr>
<tr>
<td>$\delta WS_{es}$ : site- and</td>
<td>$\phi_{SS}$</td>
<td>• Multiple recordings at individual sites</td>
</tr>
<tr>
<td>event-corrected residual</td>
<td>(aleatoric)</td>
<td></td>
</tr>
</tbody>
</table>
Example: Japanese KiKnet data, $\text{Sa}(T=0.3)$

From Rodriguez-Marek et al. (2010)
Note:

\[ \sigma_{\text{tot}} = \sqrt{\phi_{S2S}^2 + \phi_{ss}^2 + \tau^2} \]

- \( \phi_{S2S} \): site-to-site variability (after correction for site class in GMPE): **spatial**
- \( \phi_{ss} \): single-station within-event variability: **temporal**

Assumption: Standard deviation across spatial extent (across sites) is weighted equally to standard deviation across time: **ergodic assumption**
Single-station standard deviation:

\[ \sigma_{tot} = \sqrt{\phi_{S2S}^2 + \phi_{SS}^2 + \tau^2} \]

- At a single site, \( \delta S2S_s \) is predictable (deterministic)

\[ \sigma_{SS} = \sqrt{\phi_{SS}^2 + \tau^2} \]

- \( \sigma_{SS} \): single-station standard deviation

Since ergodic assumption is removed:

“Partially non-ergodic”
MOTIVATION FOR USE OF SINGLE-STATION SIGMA
Motivation for partially-ergodic PSHA

• Single-station within-event standard deviation ($\phi_{ss}$) is less variable across regions than its ergodic counterpart ($\phi$)
  – Use of global datasets?
Data from various tectonic regions

From Rodriguez-Marek et al. (2010)
Motivation for partially-ergodic PSHA

• Single-station within-event standard deviation ($\phi$) is less variable across regions than
  – Use of global datasets?

• Site response analyses:
  – An exercise in estimating $\delta S2S_s$ (and its uncertainty)
  – If $\phi_{S2S}$ is not removed: **double counting uncertainty**
Cost of partially ergodic PSHA

• Any term for which ergodicity is removed must be estimated

\[ \sigma_{tot} = \sqrt{\phi_{SS}^2 + \phi_{SS}^2 + \tau^2} \]

To do this:

We must estimate \( \delta S2S_S \) (implies also estimate of its epistemic uncertainty)
PROPOSED $\sigma_{ss}$ MODEL: DATABASES USED
Project background

• Funded by the PEGASOS PRP project
• Task: compute estimates of single station sigma
  – Data provided by Resource Experts (GMPE developers)
  – Data selection criteria (based on preliminary analyses)
    • At least 5 records per station
    • At least 5 stations per earthquake
  – Residuals were computed by individual GMPE developers
    • Single-station sigma computed only from subsets of the data used for GMPEs
  – Magnitude and distance range of interest:
    • M >= 4.5, R <= 200 km
    • Not a constraint on data for GMPE development
Preliminary observations

• Decomposition into within-event and between event (intra- and inter-event) residuals is possible
  – No correlation between $\delta B_e$ and $\delta W_{es}$

• $\tau$ estimates from regional dataset are more stable (e.g., no restriction on records per station)
  – Develop only the intra-event component ($\phi_{ss}$)

$$\sigma_{ss} = \sqrt{\phi_{ss}^2 + \tau^2}$$
Residuals were provided by ...

- California: N. Abrahamson
  - Chiou et al. California SMME database (ShakeMap)
  - Abrahamson and Silva NGA
- Japan: Rodriguez-Marek et al. (2011)
  - KiKnet data as processed by Pousse et al. (2004)
- Switzerland: Ben Edwards and Linda Al-Atik
- Taiwan: N. Abrahamson
  - Lin et al. (2011)
- Turkey: S. Akkar
Number of records used in analysis

(Table 3.1)

<table>
<thead>
<tr>
<th>T (s)</th>
<th>California</th>
<th>Switzerland</th>
<th>Taiwan</th>
<th>Turkey</th>
<th>Japan</th>
<th>All Regions</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGA</td>
<td>15295</td>
<td>1635</td>
<td>832</td>
<td>19</td>
<td>4756</td>
<td>4062</td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>3148</td>
<td>28</td>
<td>4756</td>
<td>4062</td>
</tr>
<tr>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>3514</td>
<td>28</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.3</td>
<td>15295</td>
<td>1635</td>
<td>0</td>
<td>0</td>
<td>4756</td>
<td>4062</td>
</tr>
<tr>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>3145</td>
<td>28</td>
<td>4756</td>
<td>4062</td>
</tr>
<tr>
<td>1</td>
<td>15287</td>
<td>1627</td>
<td>2108</td>
<td>28</td>
<td>4753</td>
<td>4059</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4320</td>
<td>3733</td>
</tr>
</tbody>
</table>
Number of records used in analysis

(Table 3.1)

<table>
<thead>
<tr>
<th>T (s)</th>
<th>California</th>
<th>Switzerland</th>
<th>Taiwan</th>
<th>Turkey</th>
<th>Japan</th>
<th>All Regions</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGA</td>
<td>15295</td>
<td>1635</td>
<td>832</td>
<td>19</td>
<td>4756</td>
<td>4062</td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>3148</td>
<td>28</td>
<td>4756</td>
<td>4062</td>
</tr>
<tr>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>3514</td>
<td>28</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.3</td>
<td>15295</td>
<td>1635</td>
<td>0</td>
<td>0</td>
<td>4756</td>
<td>4062</td>
</tr>
<tr>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>3145</td>
<td>28</td>
<td>4756</td>
<td>4062</td>
</tr>
<tr>
<td>1</td>
<td>15287</td>
<td>1627</td>
<td>2108</td>
<td>28</td>
<td>4753</td>
<td>4059</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Database bias:
mean of within-event residuals
(natural log units)

<table>
<thead>
<tr>
<th>T (s)</th>
<th>California</th>
<th>Switzerland</th>
<th>Taiwan</th>
<th>Turkey</th>
<th>Japan</th>
<th>All Regions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\delta W$</td>
<td>$\delta W$</td>
<td>$\delta W$</td>
<td>$\delta W$</td>
<td>$\delta W$</td>
<td>$\delta W$</td>
</tr>
<tr>
<td>M,R</td>
<td>M,R</td>
<td>M,R</td>
<td>M,R</td>
<td>M,R</td>
<td>M,R</td>
<td>M,R</td>
</tr>
<tr>
<td>PGA</td>
<td>-0.011</td>
<td>-0.031</td>
<td>0.009</td>
<td><strong>0.111</strong></td>
<td>0.000</td>
<td>-0.004</td>
</tr>
<tr>
<td>0.1</td>
<td>-</td>
<td>-</td>
<td>-0.001</td>
<td><strong>0.224</strong></td>
<td>0.000</td>
<td>-0.006</td>
</tr>
<tr>
<td>0.2</td>
<td>-</td>
<td>-</td>
<td>0.000</td>
<td><strong>0.231</strong></td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.3</td>
<td>-0.005</td>
<td>-0.033</td>
<td>-</td>
<td>-</td>
<td>0.000</td>
<td>-0.004</td>
</tr>
<tr>
<td>0.5</td>
<td>-</td>
<td>-</td>
<td>-0.001</td>
<td><strong>0.130</strong></td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>1</td>
<td>-0.007</td>
<td>-0.041</td>
<td>-0.001</td>
<td><strong>0.127</strong></td>
<td>0.000</td>
<td>0.002</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
California data

• Data from:
  – Abrahamson and Silva NGA dataset
  – Small-to-moderate magnitude data used by Chiou et al. (2010), only for PGA, 0.3 s and 1.0 s (obtained from ShakeMap)

• Residuals were obtained by separately fitting the large magnitude and small-to-moderate magnitude datasets with the Abrahamson and Silva (2008) GMPE and with the Chiou et al. (2010) GMPE, respectively.
Japan

- KiK-net data processed by Pousse et al. (2005)
- Records between 1996 and October 2004 with hypocenter depth < 25 km.
- Sa only up to T=1.0 s
- The $M_{JMA}$ was converted to seismic moment magnitude using the Fukushima (1996) relationship.
- Closest distance to the rupture was assumed to correspond to hypocentral distance for small to moderate earthquakes or when the source dimensions remain unknown. For larger earthquakes, a closest distance to the fault was computed.
- GMPE developed using both records from the surface and borehole stations were used (Rodriguez-Marek et al. 2011).
  - Magnitude scaling and event terms were constrained both by surface and borehole data
- The site term uses Vs30 as the site parameter and includes only a linear amplification term.
- Regression using Random Effects
Taiwan

• Shallow earthquakes that occurred in and near Taiwan from 1992 to 2003 for R <= 200 km (Lin et al. 2011)
• The 1999 Chi-Chi earthquake was excluded from the dataset.
• Within-event and between-event residuals of the Taiwan dataset were computed using a mixed-effects algorithm with respect to a revised version of the Chiou and Youngs (2008) model.
Turkey

• Events with depths less than 30 km.
• 239 recordings, reduced to 206 at T = 3.0 s
• Distance measure is Joyner-Boore distance metric for all recordings.
  – For small events (i.e., $M_w \leq 5.5$) epicentral distance is assumed to approximate Joyner-Boore distance.
• The database consists of very few recordings with $V_{s30} > 760$ m/s
• A one-stage maximum likelihood regression method (Joyner and Boore, 1993) is employed while developing the GMPEs.
• The functional form is the same one that is presented in Akkar and Çağnan (2010).
  – Capable of capturing magnitude saturation and magnitude-dependent geometrical spreading.
  – Site effects are considered using a function of Vs30 that also includes nonlinearity.
Switzerland

- Filtered and site-corrected acceleration time series of Swiss Foreland events used in developing the stochastic ground motion model for Switzerland (Edwards et al., 2010).
- Filtered using a variable corner frequency acausal Butterworth filter and site-corrected to correspond to the Swiss reference rock condition.
- Recordings from co-located stations were removed from the dataset.
- The form of the GMPE used to fit the Swiss data is discussed in Douglas (2009) and is described as:

\[ \ln y = b1 + b2M + b3M^2 + b4 \log(Rhyp) + b5Rhypo \]

- The random effects algorithm used in regression.
PROPOSED $\phi_{ss}$ MODEL:
CHARACTERISTICS OF THE OVERALL DATASET
Lack of regional dependence for $\phi_{ss}$

Comparison of single stations within-event standard deviation ($\phi_{ss}$) and within event standard deviations ($\phi$). Shown are data for $M\leq Mc1$ and $Rc21 \leq R \leq 200$ km.
No apparent Vs-dependency for $\phi_{ss}$
Observed Trends on $\phi_{ss}$
Distance dependence for low magnitudes

\begin{align*}
\text{M}=[4.5, 5.5] & \quad \text{Distance (km)} \\
\text{M}=[5.5, 7.0] & \quad \text{Distance (km)} \\
\text{M}=[7.0, 8.0] & \quad \text{Distance (km)}
\end{align*}
Observed Trends on $\phi_{ss}$
Magnitude dependence

$R=[0\ 16]$

$R=[16\ 40]$

$R=[40\ 200]$

CA
TW
JP
Summary

• Join together all within-event residuals from all regions
• No Vs30 dependency (more later)
• Magnitude dependency important
• Distance dependency at small magnitudes
Proposed Phi Models

• Phi Model 1: Constant $\phi_{ss}$
Proposed Phi Models

- **Phi Model 1: Constant** $\phi_{ss}$
- **Phi Model 2: Distance dependent** $\phi_{ss}$

\[
\phi_{ss}(R_{rup}) = \begin{cases} 
\phi_1 & \text{for } R_{rup} < R_{c1} \\
\phi_1 + (\phi_2 - \phi_1) \left( \frac{R_{rup} - R_{c1}}{R_{c2} - R_{c1}} \right) & \text{for } R_{c1} \leq R_{rup} \leq R_{c2} \\
\phi_2 & \text{for } R_{rup} > R_{c2}
\end{cases}
\]
Proposed Phi Models

• Phi Model 3: Distance and magnitude dependent $\phi_{ss}$

$$\phi_{ss}(M, R_{rup}) = \begin{cases} 
C_1(R_{rup}) & \text{for } M < M_{c1} \\
C_1(R_{rup}) + \left( C_2(R_{rup}) - C_1(R_{rup}) \right) \left( \frac{M - M_{c1}}{M_{c2} - M_{c1}} \right) & \text{for } M_{c1} \leq M \leq M_{c2} \\
C_2(R_{rup}) & \text{for } M > M_{c2}
\end{cases}$$

$$C_1(R_{rup}) = \phi_{11} + \left( \phi_{21} - \phi_{11} \right) \left( \frac{R_{rup} - R_{c11}}{R_{c21} - R_{c11}} \right)$$

- for $R_{rup} < R_{c11}$
- for $R_{c11} \leq R_{rup} \leq R_{c21}$
- for $R_{rup} > R_{c21}$
Methodology to determine $\phi$ values

a) A Random Effects regression was run using the entire database of within-event residuals for stations with 5 or more recordings. A constant $\phi$ model was assumed. The Random Effects give the site terms ($\delta S2S_s$).

- $\delta S2S_s$ computed with records for $M < 4.5$

b) The within event residuals were corrected with the $\delta S2S_s$ term obtained in Step a

c) Phi-models were fitted to site corrected within-event residuals ($\delta WS_{es}$)
## Tabulated values of models A, B and C

<table>
<thead>
<tr>
<th>Period</th>
<th>$\phi_{ss}$</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>Rc1</th>
<th>Rc2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.46</td>
<td>0.56</td>
<td>0.45</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>0.1</td>
<td>0.45</td>
<td>0.55</td>
<td>0.44</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>0.2</td>
<td>0.48</td>
<td>0.62</td>
<td>0.47</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>0.3</td>
<td>0.48</td>
<td>0.62</td>
<td>0.47</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>0.5</td>
<td>0.46</td>
<td>0.58</td>
<td>0.45</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>1</td>
<td>0.45</td>
<td>0.54</td>
<td>0.44</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>0.41</td>
<td>0.53</td>
<td>0.40</td>
<td>16</td>
<td>36</td>
</tr>
</tbody>
</table>
Methodology to determine $\phi$ values: M-R dependent model

- Corner magnitudes were determined first using a similar approach to that described for the distance dependent model.
- Data favored a sharp drop in $\phi_{ss}$ at M=6.0
  - A linear trend was forced by imposing the higher corner $M_{c2}$ to be equal to 7
Contour plots of Likelihood function, \( C_1 \), and \( C_2 \)

**T=3.0**

- **Optimum Point**
- **Chosen Point**
Tabulated values of Model 3

<table>
<thead>
<tr>
<th>Period</th>
<th>$\phi_{11}$</th>
<th>$\phi_{21}$</th>
<th>C2</th>
<th>Mc1</th>
<th>Mc2</th>
<th>Rc11</th>
<th>Rc21</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.58</td>
<td>0.47</td>
<td>0.34</td>
<td>5.2</td>
<td>7.0</td>
<td>16</td>
<td>36</td>
</tr>
<tr>
<td>0.1</td>
<td>0.54</td>
<td>0.44</td>
<td>0.43</td>
<td>5.2</td>
<td>7.0</td>
<td>16</td>
<td>36</td>
</tr>
<tr>
<td>0.2</td>
<td>0.60</td>
<td>0.49</td>
<td>0.37</td>
<td>5.2</td>
<td>7.0</td>
<td>16</td>
<td>36</td>
</tr>
<tr>
<td>0.3</td>
<td>0.63</td>
<td>0.50</td>
<td>0.36</td>
<td>5.2</td>
<td>7.0</td>
<td>16</td>
<td>36</td>
</tr>
<tr>
<td>0.5</td>
<td>0.59</td>
<td>0.48</td>
<td>0.36</td>
<td>5.2</td>
<td>7.0</td>
<td>16</td>
<td>36</td>
</tr>
<tr>
<td>1</td>
<td>0.54</td>
<td>0.45</td>
<td><strong>0.37</strong>*</td>
<td>5.3</td>
<td>7.0</td>
<td>16</td>
<td>36</td>
</tr>
<tr>
<td>3</td>
<td><strong>0.44</strong>*</td>
<td><strong>0.37</strong>*</td>
<td><strong>0.37</strong>*</td>
<td>5.5</td>
<td>7.0</td>
<td>16</td>
<td>36</td>
</tr>
</tbody>
</table>

* Values from standard deviation of residuals within corresponding bins are used to replace the Maximum Likelihood values (to avoid larger standard deviations for longer periods)
Site-corrected within event residuals and their standard deviations for PGA. The plot on the left shows all the residuals, the red lines indicated the standard deviation of residuals within certain distance bins. The plot on the right shows only the standard deviation of residuals with the statistical error band. The blue line is the standard deviation for all the data.
Model fit: PGA
Model fit:
T=0.2s

- All Data
- 4.0<M<5.2
- 7.0<M<8.0
- 0<R<16
- 32<R<200

Magnitude vs. Distance (km) for different models.
Model fit:

\[ T = 1.0 \text{s} \]
Comparison with ‘ergodic’ values (magnitude-independent model)

<table>
<thead>
<tr>
<th>Period (s)</th>
<th>Proposed Model 2</th>
<th>Based on within-event residuals of AS08</th>
<th>Within event residuals in PRP data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\phi_{ss1}$</td>
<td>$\phi_{ss2}$</td>
<td>$\phi_1$</td>
</tr>
<tr>
<td>0.01</td>
<td>0.56</td>
<td>0.45</td>
<td>0.50</td>
</tr>
<tr>
<td>0.1</td>
<td>0.55</td>
<td>0.44</td>
<td>0.53</td>
</tr>
<tr>
<td>0.2</td>
<td>0.62</td>
<td>0.47</td>
<td>0.55</td>
</tr>
<tr>
<td>0.3</td>
<td>0.62</td>
<td>0.47</td>
<td>0.54</td>
</tr>
<tr>
<td>0.5</td>
<td>0.58</td>
<td>0.45</td>
<td>0.53</td>
</tr>
<tr>
<td>1</td>
<td>0.54</td>
<td>0.44</td>
<td>0.53</td>
</tr>
<tr>
<td>3</td>
<td>0.53</td>
<td>0.40</td>
<td>0.56</td>
</tr>
</tbody>
</table>
Al Atik (pers. Comm)
Comparison with ‘ergodic’ values (magnitude-dependent model)

<table>
<thead>
<tr>
<th>Period (s)</th>
<th>Proposed Model 3 (single station)</th>
<th>Based on within-event residuals of AS08 (ergodic)</th>
<th>Within event residuals in PRP data (ergodic)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\phi_{11}$</td>
<td>$\phi_{21}$</td>
<td>$C_2$</td>
</tr>
<tr>
<td>0.01</td>
<td>0.58</td>
<td>0.47</td>
<td>0.34</td>
</tr>
<tr>
<td>0.1</td>
<td>0.54</td>
<td>0.44</td>
<td>0.43</td>
</tr>
<tr>
<td>0.2</td>
<td>0.60</td>
<td>0.49</td>
<td>0.37</td>
</tr>
<tr>
<td>0.3</td>
<td><strong>0.63</strong></td>
<td>0.50</td>
<td>0.36</td>
</tr>
<tr>
<td>0.5</td>
<td><strong>0.59</strong></td>
<td>0.48</td>
<td>0.36</td>
</tr>
<tr>
<td>1</td>
<td><strong>0.54</strong></td>
<td>0.45</td>
<td>0.37</td>
</tr>
<tr>
<td>3</td>
<td>0.44</td>
<td>0.37</td>
<td>0.37</td>
</tr>
</tbody>
</table>
Al Atik (pers. Comm)
VARIATION OF $\phi_{ss}$ FROM STATION TO STATION
Epistemic uncertainty on $\phi_{ss}$

- Both the site term ($\delta S2S_s$) and the single-station standard deviation ($\phi_{ss}$) have epistemic uncertainties.
  - Different stations sample different sources/paths
  - 2D and 3D effects imply azimuthal dependency
  - Degree of nonlinearity
Azimuthal dependency (KiK-net data)

T = 1 s
Stations with N >= 15
$N_{\text{min}} = 10$

T=0.0, All Sites $M_{\geq 4.5}$, $R_{\leq 200}$, $N_{\text{min}}=10$

- Normal
  - $\chi^2 = 1$
  - $K-S = 0$
  - Lillie = 1

- Log Normal
  - $\chi^2 = 0$
  - $K-S = 0$
  - Lillie = 0

Goodness of fit test ($1$ = reject)

$\phi = 0.57$
$\phi_{ss} = 0.46$
$\phi_{ss,s} = 0.86$
$\min \phi_{ss,s} = 0.20$
$\phi_{ss,s} = 0.43$
$\max \phi_{ss,s} = 0.86$
$\phi_{ss,s} = 0.20$

$\text{mean } \phi_{ss,s} = 0.43$
$\text{std } \phi_{ss,s} = 0.10$
Nmin = 15

\[ T=0.0, \text{All Sites M} \geq 4.5, \text{R} \leq 200, \text{Nmin}=15 \]

\[ \phi = 0.57 \]
\[ \phi_{ss} = 0.46 \]
\[ \text{max } \phi_{ss,s} = 0.69 \]
\[ \text{min } \phi_{ss,s} = 0.23 \]
\[ \text{mean } \phi_{ss,s} = 0.44 \]
\[ \text{std } \phi_{ss,s} = 0.09 \]
Nmin = 20

T=0.0, All Sites M>=4.5, R<=200, Nmin=20

Goodn. of fit test (1=reject)

Normal
JB=0
KS=0
Lillie=0

Log Normal
JB=0
KS=0
Lillie=0

φ = 0.57
φss = 0.46
max φss,s = 0.63
min φss,s = 0.28
mean φss,s = 0.44
std φss,s = 0.08
### Variation in $\phi_{ss,s}$

<table>
<thead>
<tr>
<th>Period (s)</th>
<th>$N \geq 10$</th>
<th></th>
<th></th>
<th>[N \geq 15]</th>
<th></th>
<th></th>
<th>[N \geq 20]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>Stations</td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>Stations</td>
<td>Mean</td>
<td>Std. Dev.</td>
</tr>
<tr>
<td>0.01</td>
<td>0.43</td>
<td>0.10</td>
<td>326</td>
<td>0.44</td>
<td>0.09</td>
<td>133</td>
<td>0.44</td>
<td>0.08</td>
</tr>
<tr>
<td>0.1</td>
<td>0.45</td>
<td>0.12</td>
<td>316</td>
<td>0.45</td>
<td>0.10</td>
<td>133</td>
<td>0.45</td>
<td>0.08</td>
</tr>
<tr>
<td>0.2</td>
<td>0.47</td>
<td>0.12</td>
<td>50</td>
<td>0.52</td>
<td>0.10</td>
<td>13</td>
<td>0.56</td>
<td>0.11</td>
</tr>
<tr>
<td><strong>0.3</strong></td>
<td><strong>0.46</strong></td>
<td><strong>0.11</strong></td>
<td><strong>326</strong></td>
<td><strong>0.47</strong></td>
<td><strong>0.10</strong></td>
<td><strong>133</strong></td>
<td><strong>0.47</strong></td>
<td><strong>0.09</strong></td>
</tr>
<tr>
<td>0.5</td>
<td>0.46</td>
<td>0.11</td>
<td>316</td>
<td>0.47</td>
<td>0.09</td>
<td>133</td>
<td>0.46</td>
<td>0.08</td>
</tr>
<tr>
<td>1</td>
<td>0.44</td>
<td>0.10</td>
<td>326</td>
<td>0.43</td>
<td>0.08</td>
<td>133</td>
<td>0.43</td>
<td>0.07</td>
</tr>
<tr>
<td>3</td>
<td>0.41</td>
<td>0.10</td>
<td>245</td>
<td>0.42</td>
<td>0.08</td>
<td>89</td>
<td>0.41</td>
<td>0.07</td>
</tr>
</tbody>
</table>
Variation in range of $\phi_{ss,s}$

<table>
<thead>
<tr>
<th>Period (s)</th>
<th>$N \geq 10$</th>
<th></th>
<th></th>
<th>$N \geq 15$</th>
<th></th>
<th></th>
<th>$N \geq 20$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Min</td>
<td>Max</td>
<td>Mean</td>
<td>Min</td>
<td>Max</td>
<td>Mean</td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>0.01</td>
<td>0.43</td>
<td>0.20</td>
<td>0.86</td>
<td>0.44</td>
<td>0.23</td>
<td>0.69</td>
<td>0.44</td>
<td>0.28</td>
<td>0.63</td>
</tr>
<tr>
<td>0.1</td>
<td>0.45</td>
<td>0.21</td>
<td>0.85</td>
<td>0.45</td>
<td>0.22</td>
<td>0.77</td>
<td>0.45</td>
<td>0.25</td>
<td>0.67</td>
</tr>
<tr>
<td>0.2</td>
<td>0.47</td>
<td>0.25</td>
<td>0.79</td>
<td>0.52</td>
<td>0.37</td>
<td>0.65</td>
<td>0.56</td>
<td>0.37</td>
<td>0.65</td>
</tr>
<tr>
<td>0.3</td>
<td>0.46</td>
<td>0.19</td>
<td>0.87</td>
<td>0.47</td>
<td>0.27</td>
<td>0.73</td>
<td>0.47</td>
<td>0.31</td>
<td>0.63</td>
</tr>
<tr>
<td>0.5</td>
<td>0.46</td>
<td>0.17</td>
<td>0.98</td>
<td>0.47</td>
<td>0.25</td>
<td>0.70</td>
<td>0.46</td>
<td>0.31</td>
<td>0.61</td>
</tr>
<tr>
<td>1</td>
<td>0.44</td>
<td>0.19</td>
<td>0.93</td>
<td>0.43</td>
<td>0.27</td>
<td>0.66</td>
<td>0.43</td>
<td>0.27</td>
<td>0.60</td>
</tr>
<tr>
<td>3</td>
<td>0.41</td>
<td>0.17</td>
<td>0.89</td>
<td>0.42</td>
<td>0.25</td>
<td>0.68</td>
<td>0.41</td>
<td>0.30</td>
<td>0.54</td>
</tr>
</tbody>
</table>
Note: possible Vs30 dependency? (KiK-net data)
Epistemic Uncertainty on $\phi_{ss}$

Summary

- There is variability of $\phi_{ss}$ from station to station: Epistemic uncertainty
- The standard deviation of $\phi_{ss,s}$ can only be estimated for a constant Phi Model (Model 1)
  - Standard deviation of $\phi_{ss,s}$ reduces as N increases
  - Range of $\phi_{ss,s}$ reduces as N increases
- Can’t reject normality for distribution of $\phi_{ss,s}$
Epistemic Uncertainty on $\phi_{ss}$

Summary

• The standard deviation of the single-station phi measured at each station $[\text{std}(\phi_{ss,s})]$ can be used as a basis to assign epistemic uncertainty on $\phi_{ss}$
  – Use well recorded stations ($N \geq 20$) to estimate $\text{std}(\phi_{ss,s})$
  – Strictly applicable only to constant phi model
  – Applicable to other phi models? [some of the variability may be due to M,R variability]
Notes: Consistency with other studies
Notes: Consistency across regions

\( \phi_{ss} \)

<table>
<thead>
<tr>
<th></th>
<th>CA</th>
<th>Taiwan</th>
<th>Japan</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGA</td>
<td>.51</td>
<td>.51</td>
<td>.50</td>
<td>.51</td>
</tr>
<tr>
<td>T=0.3</td>
<td>.54</td>
<td>.51</td>
<td>.50</td>
<td>.52</td>
</tr>
<tr>
<td>T=1.0s</td>
<td>.47</td>
<td>.47</td>
<td>.44</td>
<td>.46</td>
</tr>
</tbody>
</table>

\( \phi_{S2S} \)

<table>
<thead>
<tr>
<th></th>
<th>CA</th>
<th>Taiwan</th>
<th>Japan</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGA</td>
<td>.41</td>
<td>.25</td>
<td>.48</td>
<td>.40</td>
</tr>
<tr>
<td>T=0.3</td>
<td>.44</td>
<td>.28</td>
<td>.46</td>
<td>.43</td>
</tr>
<tr>
<td>T=1.0s</td>
<td>.44</td>
<td>.37</td>
<td>.37</td>
<td>.41</td>
</tr>
</tbody>
</table>

- Note
  - Consistency of \( \phi_{ss} \) across regions
  - \( \phi_{S2S} \) is more variable across regions
## Proposed $\phi_{ss}$ models

<table>
<thead>
<tr>
<th>$\phi_{ss}$ Model</th>
<th>Epistemic Uncertainty on $\phi_{ss}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phi Model 1 (Constant Phi)</td>
<td></td>
</tr>
<tr>
<td>Phi Model 2 (Distance Dependent) – Overpredicts at large magnitudes</td>
<td>Consider standard deviation in $\phi_{ss,s}$</td>
</tr>
<tr>
<td>Phi Model 3 (Distance- and Magnitude dependent)</td>
<td></td>
</tr>
</tbody>
</table>
Conclusions

• $\tau$ must be added
  • Proponent position: consider tau estimates independently

• Conditions for application of single-station sigma must be met
  • Estimate of site term ($\delta S2S_s$) and its epistemic uncertainty
  • If not, must use total sigma

\[
\sigma_{tot} = \sqrt{\phi_{S2S}^2 + \phi_{SS}^2 + \tau^2}
\]

• Proponent position: estimates of $\phi_{S2S}$ must also be given
Thank you
California data

- The California dataset used in this study consists of ground motion data from the Abrahamson and Silva (2008) NGA dataset and small-to-moderate magnitude California data used for the small magnitude extension of the Chiou and Youngs (2008) NGA model (Chiou et al., 2010) at peak ground acceleration (PGA) and at periods of 0.3 and 1 second. The Abrahamson and Silva (2008) data are part of the NGA West dataset. The small-to-moderate magnitude data used in Chiou et al. (2010) were obtained from ShakeMap.

- Between-event residuals and within-event residuals of these two California datasets were obtained by separately fitting the large magnitude and small-to-moderate magnitude datasets with the Abrahamson and Silva (2008) GMPE and with the Chiou et al. (2010) GMPE, respectively.
Switzerland

- The Swiss dataset used in this study consists of filtered and site-corrected acceleration time series of Swiss Foreland events used in developing the stochastic ground motion model for Switzerland (Edwards et al., 2010). The acceleration time series were filtered using a variable corner frequency acausal Butterworth filter and site-corrected to correspond to the Swiss reference rock condition. A detailed description of the Swiss dataset is given in Edwards et al. (2010). Recordings from co-located stations were removed from the dataset and response spectra of the remaining recordings were computed using the Nigam and Jennings (1969) algorithm. Only recordings from events with more than 5 recordings and at stations with more than 5 recordings were used in this analysis.

- The form of the GMPE used to fit the Swiss data is discussed in Douglas (2009) and is described as:

\[ \ln y = b1 + b2M + b3M^2 + b4 \log(Rhypo) + b5Rhypo \]

- The random effects algorithm developed by Abrahamson and Youngs (1992) was used to calculate the parameters of the GMPE fit to the Swiss dataset and to obtain the between-event and within-event residuals.
Japan

- The Japanese data used in this study is data downloaded from the KiK-net network website and is described in Pousse et al. (2005), Pousse et al. (2006), and Cotton et al. (2008). Only records between 1996 and October 2004 were used. In addition, only records with hypocenter depth less than 25 km were used in order to avoid subduction related events.

- The $M_{JMA}$ was converted to seismic moment magnitude using the Fukushima (1996) relationship (Cotton et al. 2008). Data processing is described in Pousse et al. (2005) and Cotton et al. (2008). Cotton et al. (2008) states that the longest usable period for the data set is 3.0 s. However, some of the spectral accelerations at long periods are lower than the number of decimals in the data set. For that reason, only spectral periods up to 1.0 s are used in this work. Closest distance to the rupture was assumed to correspond to hypocentral distance for small to moderate earthquakes or when the source dimensions remain unknown. For larger earthquakes, a closest distance to the fault was computed.

- A peculiarity in the development of the GMPE for the KiK-net data is that both records from the surface and borehole stations were used (with appropriate site terms). This implies that the event-terms and magnitude scaling was constrained both by surface and borehole data (Rodriguez-Marek et al. 2011). The site term uses $V_{s30}$ as the site parameter and includes only a linear amplification term. Parameters of the GMPE equation were obtained using the Random Effects regression of Abrahamson and Youngs (1992)
Taiwan

- The Taiwan dataset used in this study consists of ground motion data from shallow earthquakes that occurred in and near Taiwan from 1992 to 2003. This dataset was assembled by Lin (2009) and the ground motion recordings were baseline corrected and limited to distances of less than 200 km. The 1999 Chi-Chi earthquake was excluded from the dataset. A more detailed description of the Taiwan data is given in Lin et al. (2011). Within-event and between-event residuals of the Taiwan dataset were computed using a mixed-effects algorithm with respect to a revised version of the Chiou and Youngs (2008) model. Only stations with at least 10 recordings were used to calculate the site terms.
Turkey

- The Turkish data used in this study is compiled within the framework of the project entitled “Compilation of Turkish strong-motion network according to the international standards.” The procedures followed to assemble the database are described in Akkar et al. (2010) and Sandıkkaya et al. (2010). The dataset comprises of events with depths less than 30 km. Each event in the dataset has at least 5 recordings. The distance measure is Joyner-Boore distance metric for all recordings. For small events (i.e., $M_w \leq 5.5$) epicentral distance is assumed to approximate Joyner-Boore distance. The dataset is processed (band-pass acausal filtering) by the method described in Akkar and Bommer (2006). The same article also describes a set of criteria for computing the usable spectral period range that is used to define the maximum usable period for each processed record. The database consists of very few recordings with $V_{s30} > 760$ m/s (where $V_{s30}$ is the average shear wave velocity over the upper 30 m of a profile) and almost all events are either strike-slip or normal. The database contains 239 recordings but this number is reduced to 206 at $T = 3.0$ s due usable period range criteria in Akkar and Bommer (2006).

- Regression analysis is conducted using the above dataset to derive a set of GMPEs for the analysis of within event residuals. A one-stage maximum likelihood regression method (Joyner and Boore, 1993) is employed while developing the GMPEs. The model estimates the geometric mean of the ground motion. The functional form is the same one that is presented in Akkar and Çağnan (2010). It is capable of capturing magnitude saturation and magnitude-dependent geometrical spreading. Site effects are considered using a function of $V_{s30}$ that also includes nonlinearity.